Analysis of an Inhomogeneously Loaded Rectangular Waveguide with Dielectric and Metallic Losses

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Abstract—**A dielectric loaded rectangular guide (Fig. 1) is analyzed taking into account the losses of the line, the substrate, and the metallic shield. For this study, Schelkunoff's method [1] is used in considering the propagation modes as a convenient combination of coupled modes of the empty waveguide. Computations and experimental measurements are made for two different lines.**

I. Introduction

The METHOD formulated by Schelkunoff avoids writing the boundary conditions of the different dielectric media contained in the guide. The line studied is represented by an infinite set of coupled transmission lines, a propagation mode of the empty shield being assigned to each one. In our case, these modes are those of the empty rectangular waveguide. Maxwell's equations are written under an expanded form with insertion of the field expressions. After an analytical computation, generalized telegraphists' equations are obtained. These equations, given in [1]–[3] permit the computation of the phase constant β and the attenuation constant α_d due to the dielectric.

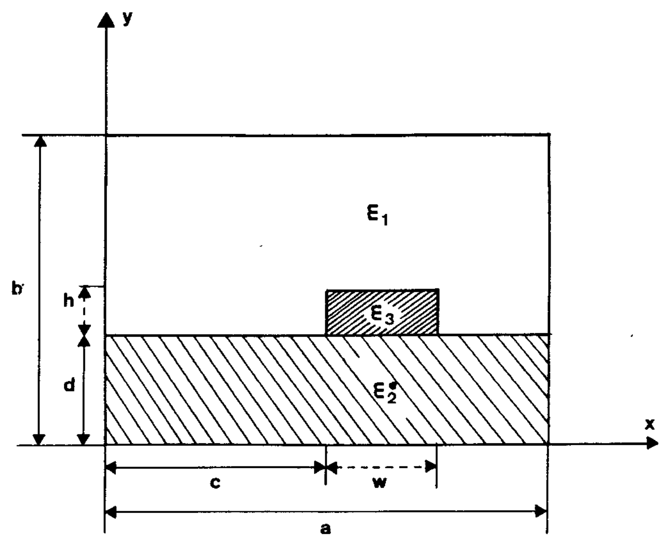

Fig. 1. Transversal section of the shielded dielectric line.

II. Phase Constant β

A. Theory

For an exact solution, it would be necessary to consider an infinite number of modes. However a good approximation is obtained by considering only the modes that are most strongly coupled to the fundamental one. Several authors [2]–[5] have recently used this method of analysis with good results for a two-mode approximation and a three-mode one. The dielectric line commonly used has small dimensions and a low dielectric constant. For better precision, particularly in the case of dielectric lines with high ε_r, it is necessary to consider a higher order approximation. Obviously the study becomes more and more difficult as the number of modes increases.

The shielded line is a rectangular waveguide ($a \neq 2b$) asymmetrically loaded by a high dielectric constant slab which may be put anywhere (Fig. 1). The dielectric substrate $\varepsilon_2(\varepsilon_2 < \varepsilon_3)$ under the line considered is used only as a support.

In this work, a 15-mode approximation has been chosen. These 15 modes (ten TE modes and five TM modes) of the empty guide have been selected according to the order of their cut-off frequency.

If the bar is centered in the guide, the structure has a symmetrical axis (0y axis put in $x = a/2$) and the modes may be classified into two kinds: the symmetrical modes and the antisymmetrical ones which are not having coupling between them. The new 0y axis (in $x = a/2$, Fig. 1) is a magnetic wall ($H_z = 0$) for a symmetrical mode such as $TE_{10}, TE_{11}, TM_{11}, TE_{30}, \cdots$, and an electric wall ($E_z = 0$) for an antisymmetrical mode such as TE_{20}, TE_{01}, TE_{21}, $TM_{21}, \cdots$. So, in order to study the fundamental mode (pseudo TE_{10}) of a symmetrical line, it will be necessary to choose the first 15 symmetrical modes. With this supplementary condition in the selection of modes, the results for a symmetrical line will be more accurate than with an asymmetrical one.

From this approximation one can write down a set of 30 telegraphist's equations which will not have roots unless their determinant is zero. This condition allows one to compute the propagation constant $\Gamma = \alpha_d + j\beta$.

B. Results

The values of β and α_d are computed between 2 and 4 GHz for two different dielectric lines:

$$\varepsilon_3 = 13(1 - j2 \times 10^{-4}) \quad \text{(D-13)}$$

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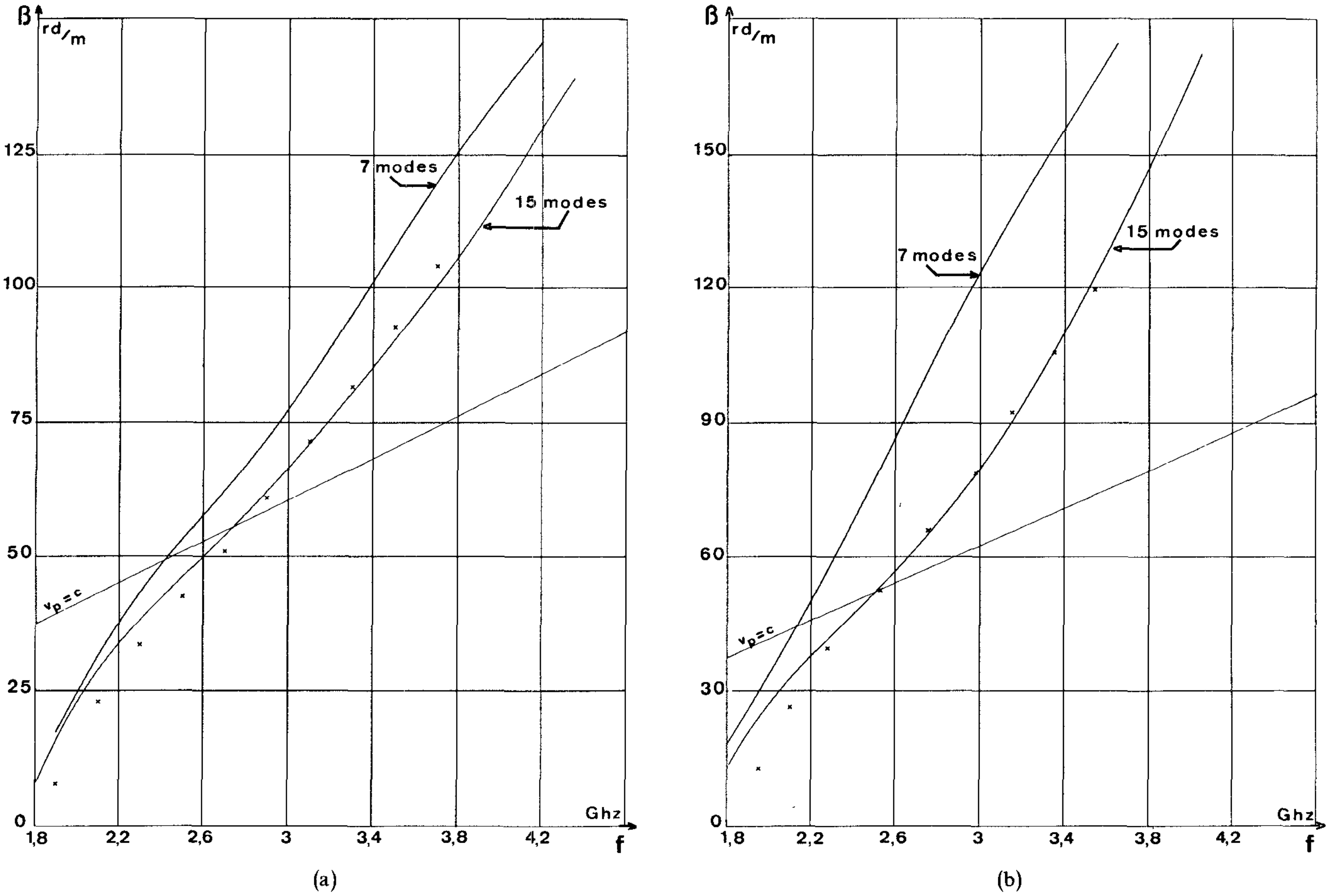

Fig. 2. Phase constant versus frequency for the fundamental mode: ———: theoretical results; xxxxx: experimental results. (a) $\varepsilon_3 = 13$, $c/a = 0.163$, $w/a = 0.346$, $d/b = 0.093$, $h/b = 0.288$. (b) $\varepsilon_3 = 30$, $c/a = 0.161$, $w/a = 0.354$, $d/b = 0.093$, $h/b = 0.235$

and

$$\varepsilon_3 = 30(1 - j2 \times 10^{-3}) \quad \text{(MCT—30 Trans Tech.)}$$

with

$$\varepsilon_1 = 1 \quad \text{(air)} \quad \text{and} \quad \varepsilon_2 = 2.54(1 - j10^{-4}) \quad \text{(rexolite)}$$

$$a = 72.1 \text{ mm} \qquad b = 34 \text{ mm}$$

$$d = 3.2 \text{ mm} \qquad w = h = 10 \text{ mm}.$$

Fig. 2(a) and (b) exhibits good agreement between theory and experiment for the fundamental mode with a 15-mode approximation and shows that a seven mode approximation is insufficient. It is possible to draw the dispersion curves of the 15 modes but the precision is only good for the first three or four higher modes (Fig. 3(a) and (b)).

III. Attenuation Constants: α_d and α_m

A. Theory

Since the losses are less, one is quite justified in assuming that they do not disturb the field geometry, and then the metallic and dielectric losses may be considered independently.

When the determinant is zero, the attenuation constant α_d can be computed, and then it is possible to find the solution of the set of equations ("voltage" and "intensity" of each mode) and, hence, to find the components of the electric and magnetic fields. It is easy to compute P_J, the power dissipated by Joule heating in the shield and W_{ac} the energy accumulated in the line:

$$P_J = \tfrac{1}{2} R_S \oint \bar{H}_{tg} \cdot \bar{H}_{tg}^{*} \, dl \quad \text{(guide walls)} \tag{1}$$

when

$$W_{ac} = \tfrac{1}{2} \iint_S \left(\frac{\mu}{2} |\bar{H}|^2 + \frac{\varepsilon}{2} |\bar{E}|^2 \right) ds. \tag{2}$$

The above relations allow the calculation of α_m

$$\alpha_m = \frac{1}{2} \frac{P_J}{v_g \cdot W_{ac}} \qquad \left(v_g = \frac{d\omega}{d\beta} : \text{group velocity} \right). \tag{3}$$

The conductivity σ of the copper shield used in the experiment has been measured. So the value $\sigma = 7.7 \times 10^6$ MKS has been used in the computation instead of $\sigma = 5.8 \times 10^7$ MKS, which is the optimum value for copper with a perfectly polished surface.

B. Results

Fig. 4 shows the whole of the losses $\alpha_t = \alpha_d + \alpha_m$ for the fundamental mode. Table I gives some indications.

Theory and experiment are in good agreement. The loss

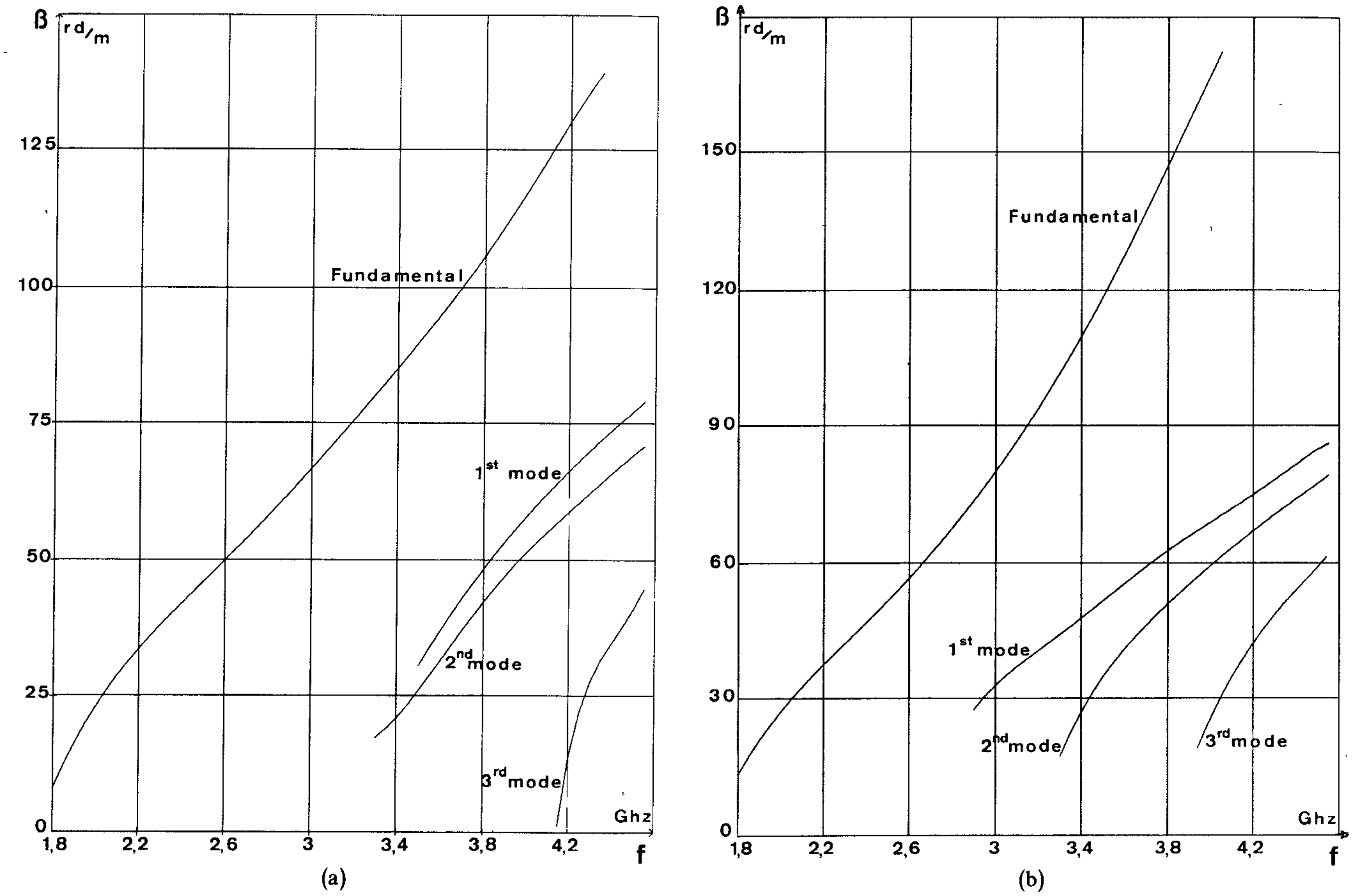

Fig. 3. Phase constant versus frequency for the fundamental and first three modes. (a) $\varepsilon_3 = 13$. (b) $\varepsilon_3 = 30$.

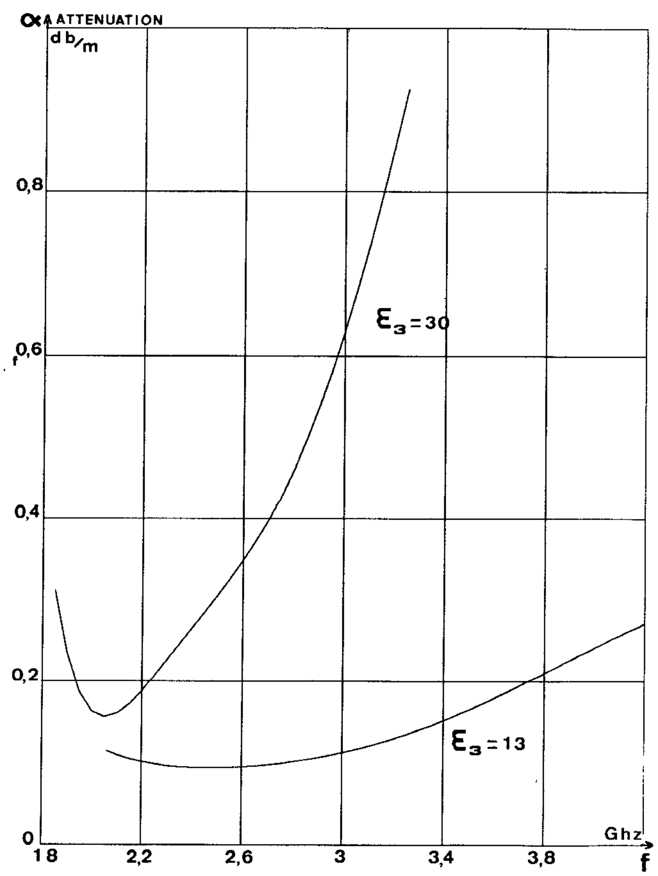

Fig. 4. Attenuation constant versus frequency for the fundamental mode.

TABLE I

ε_3	frequency (GHz)	losses: theory db/m	losses: experiment db/m	losses: empty shield db/m
13(± 5 %)	2,4	α_m = 0,07 α_d = 0,027 α_t = 0,097	α_t = 0,12	α = 0,078
	3	α_m = 0,066 α_d = 0,046 α_t = 0,112	α_t = 0,14	α = 0,051
30(± 5 %)	3	α_m = 0,08 α_d = 0,53 α_t = 0,61	α_t = 0,56	α = 0,051

tangent used in the computation is the maximum value given by the dielectric material manufacturer.

It is also necessary to point out that losses have not been minimized either by using a polished shield and better dielectric materials or by trying to find the best dimensions and position of the dielectric line.

Conclusion

This work shows that the Schelkunoff's method gives good results provided the number of considered modes is adequate. This method is powerful and its use is certainly necessary in more complex cases such as coupled lines, anisotropic lines, etc.

The elaborated computed program is very general. It is very easy to increase the number of modes, to choose structures with n dielectric media, and to analyze anisotropic lines.

ACKNOWLEDGMENT

The computations used for this analysis have been made in the Computer Center of the University of Limoges.

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Dispersion in Shielded Planar Transmission Lines on Two-Layer Composite Substrate

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Abstract—**The singular integral equation technique has been used to analyze a shielded planar transmission line, which allows one to calculate the dispersion characteristics of shielded microstrips on two-layer substrates as well as the effect of shielding on coplanar waveguides. Dispersion curves for suspended substrate microstrips and the variation of the relative phase velocity, with frequency, of coplanar waveguide (CPW) on alumina substrates of finite thicknesses and variable ground plane positions are presented. The results of computations with the lowest order 4 × 4 determinant show good agreement with the available data.**

I. INTRODUCTION

THE theoretical analysis as well as the experimental investigation of the frequency dependent behavior of various planar transmission lines has received considerable attention in recent years [1]–[9]. These include shielded and unshielded microstrip lines and their derivatives such as slot lines, CPW, and coplanar strips on both infinite and finite dielectric substrates. However, not much information is available for dispersion in shielded microstrips on composite substrates. Hasegawa [10] and Guckel [11] have studied the open microstrips on composite substrates of finite conductivity using the parallel-plate waveguide model. Krage and Haddad [4] have presented the dispersion characteristics of single and coupled microstrips with overlay dielectric and of coupled microstrips on composite substrate, but, unfortunately, have not given any data on single microstrip on two-layer substrate. This author has used the singular integral equation technique of Mittra and Itoh [6] to study the dispersion characteristics of shielded single microstrips on composite substrates. The structure has been further modified by introducing two symmetrical coplanar ground planes (Fig. 1) so that the same analysis enables one to calculate the effect of shielding on dispersion in CPW on a finite substrate. Since the completion of the present work the author came across the recent work of Yamashita and Atsuki [12], who have analyzed similar structures by nonuniform discretization of integral equations.

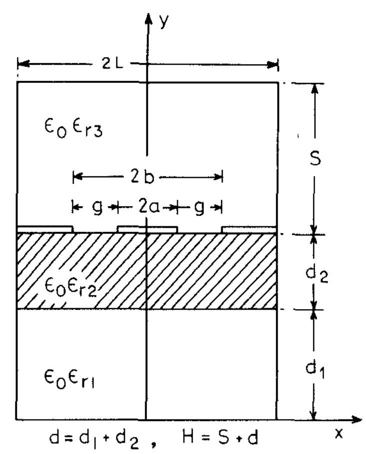

Fig. 1. Shielded coplanar waveguide.

Since the basic technique is the same as in [6], many steps in the mathematical derivation are omitted, and only the new relations necessary for computation are given explicitly. Results are presented in the form of normalized dispersion curves for the dominant modes only, although the higher

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